

Dirac-Born-Infeld-Volkov-Akulov and Deformation of Supersymmetry

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Abstract

We deform the action and the supersymmetry transformations of the $d = 10$ and $d = 4$ Maxwell supermultiplets so that at each order of the deformation the theory has 16 Maxwell multiplet deformed supersymmetries as well as 16 Volkov-Akulov type non-linear supersymmetries. The result agrees with the expansion in the string tension of the explicit action of the Dirac-Born-Infeld model and its supersymmetries, extracted from D9 and D3 superbranes, respectively. The half-maximal Dirac-Born-Infeld models with 8 Maxwell supermultiplet deformed supersymmetries and 8 Volkov-Akulov type supersymmetries are described by a new class of $d = 6$ vector branes related to chiral (2,0) supergravity, which we denote as ‘Vp-branes’. We use a space-filling V5 superbrane for the $d = 6$ model and a V3 superbrane for the $d = 4$ half-maximal Dirac-Born-Infeld (DBI) models. In this way we present a completion to all orders of the deformation of the Maxwell supermultiplets with maximal 16+16 supersymmetries in $d = 10$ and 4, and half-maximal 8+8 supersymmetries in $d = 6$ and 4.

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1 Introduction

The purpose of this study is to look for new ways of constructing supersymmetric invariants for theories with extended supersymmetries where there are no known auxiliary fields. For example, the supersymmetries of a $d = 4$, $\mathcal{N} = 4$ Maxwell multiplet form an open algebra, it is closed only when the classical equations of motion are satisfied. This prevents the direct use of the $\mathcal{N} = 4$ superconformal tensor calculus to construct superconformal invariants with higher derivatives [1]. It is different from the $\mathcal{N} = 2$ case, where some superconformal multiplets form a closed algebra, the auxiliary fields are known, and higher derivative superconformal invariants can be constructed using superconformal calculus, as shown in detail in [2, 3]. When auxiliary fields are eliminated using their equations of motion, one finds a deformed local $\mathcal{N} = 2$ supersymmetry and the deformed $\mathcal{N} = 2$ supergravity action (after gauge fixing of extra symmetries), which depend only on physical fields [3].

Here we will build the higher derivative supersymmetric gauge theory model developing the proposal in [4] to deform the quadratic action of the $d = 10$ Maxwell multiplet where the deformation parameter is the open string tension. We refer the reader to Born-Infeld and Dirac models [5, 6], its supersymmetric generalizations and its relation to string theory discussed in [7–22]. A superembedding approach as a generic covariant method for the description of superbranes as models of partial spontaneous supersymmetry breaking was developed in [23, 24]. In the context of extended $d = 4$ supergravity and duality symmetry there is a significant interest to Born-Infeld type constructions [25–31].

Our work will consist of bottom up and top down deformation of the supersymmetric Maxwell action:

$$S = \int d^{10}x \left\{ -\frac{1}{4}(F_{\mu\nu})^2 + \bar{\lambda}\not{\partial}\lambda \right\}. \quad (1.1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. There is no known off-shell formulation of this multiplet. The same applies to the $\mathcal{N} = 4$ version of it in $d = 4$ where our current goal is to learn some new ways of building higher derivative actions involving this supermultiplet [1].

This theory describes $8 + 8$ on-shell degrees of freedom and consists of a vector A_μ and a 16-component Majorana-Weyl spinor λ . The on-shell 16 linear supersymmetries are given by

$$\delta_\epsilon A_\mu = \bar{\epsilon}\Gamma_\mu\lambda, \quad \delta_\epsilon\lambda = \frac{1}{4}\Gamma^{\mu\nu}F_{\mu\nu}\epsilon. \quad (1.2)$$

The action (1.1) has also a trivial fermionic symmetry under which the fermion shifts by a constant parameter

$$\delta_\eta A_\mu = 0, \quad \delta_\eta\lambda = -\frac{1}{2\alpha}\eta. \quad (1.3)$$

In the bottom up procedure we will deform the Maxwell action and the 16 supersymmetries of the linear action order by order in the string tension, so that at each order the deformed action has the

symmetries (1.2), deformed by $\mathcal{O}(\alpha)$ terms, as well as 16 hidden Volkov-Akulov type supersymmetries [32] of the form

$$\delta_\zeta A_\mu = -\bar{\zeta}\Gamma_\mu\lambda + \mathcal{O}(\alpha), \quad \delta_\zeta\lambda = \alpha^{-1}\zeta + \mathcal{O}(\alpha). \quad (1.4)$$

We will compare this with the top down approach based on the D9 super-brane action, [33–38]. It has been noticed in [12] that the supersymmetry of the D9 super-brane action, upon gauge-fixing of a local κ -symmetry, has a complicated form. This may have been one of the reasons that in $d = 4$ a better understanding of Dirac-Born-Infeld (DBI) type models was achieved in the past by developing the superfield models where, for example, the linear $\mathcal{N} = 2$ off shell supersymmetry including auxiliary fields was manifest, whereas the non-linear spontaneously broken one was deformed and often called ‘hidden’ [10–13, 15–18, 23–27].

Here we will present an explicit, relatively simple and complete form of the $d = 10$ DBI model with 16 deformed supersymmetries of the Maxwell multiplet and 16 Volkov-Akulov (VA) type non-linear supersymmetries. The original VA model was proposed in $d = 4$ in [32]. The $d = 10$ analog was discussed in the context of the D9 branes in [38]. The details of the derivation of this model from the κ -symmetric D9-superbrane action are presented in Appendix A. It will also be shown that all these symmetries of the DBI-VA model can be expanded in the string tension deformation parameter and this expansion coincides with the bottom up model of the deformation of the Maxwell multiplet. Thus, there is a *known completion of the deformation process* and the action as well as all non-linear 16 + 16 supersymmetries can be given in the compact form.¹ Using the D3 super-brane we will also present a complete and explicit $d = 4$ DBI model with maximal number of non-linear supersymmetries, half being VA-type. This new model with 16+16 supersymmetries in $d = 4$ could be called $\mathcal{N} = 8$ DBI, by analogy with the $\mathcal{N} = 4$ DBI model in [18] with 8+8 supersymmetries. No such DBI model was constructed before.

To study the half-maximal supersymmetric DBI models with 8 deformed supersymmetries and 8 VA-type supersymmetries of the Maxwell multiplet, we will introduce a new class of $d = 6$ ‘vector branes’ whose world-volume dynamics is described by a vector multiplet but whose tension does not necessarily scale with the inverse string coupling constant, like it is the case for Dirichlet branes. Such branes are suggested by a recent analysis of the different branes, and their world-volume content, of the $d = 6$ half-maximal theories [39]. For our purposes it is sufficient to consider the vector branes related to $d = 6$ (2,0) chiral supergravity.

¹We will use the terminology 16 + 16 supersymmetry. The first term ‘16’ refers to the number of supersymmetries that have a linear part and imply equal number of bosons and fermions and existence of the particle representations. The second term ‘16’ refers to non-linear supersymmetries of the VA type, that do not determine the particle content. The same holds for 8 + 8 case.

As inherited from the supersymmetric DBI model in $d = 10$ and $d = 6$, the supersymmetric DBI models in $d = 4$ feature a complete and explicit deformation to all orders of all supersymmetries. This might help to provide an all order completion of the $\mathcal{N} = 4$ DBI model in [18] with the same number of supersymmetries.

To compare our new complete models with the ones in $d = 4$, [8, 10–13, 15–18], will require a separate investigation. In these models, half of the supersymmetry is manifest in superspace; it involves the auxiliary fields and it is not deformed order by order (only the hidden supersymmetry is deformed). The action of these models with 8 supersymmetries manifest and 8 hidden supersymmetries is known only up to order 10 in superfields, as shown in [18]. There seems to be no known algorithm which would generate the action at higher orders of deformation. The actions which we will construct here, with the 16+16 and 8+8 all deformed non-linear supersymmetries, are complete and therefore will be known at every level of deformation, however none of the supersymmetries will be manifest.

This paper is organized as follows. In section 2 we present the DBI action with 16+16 supersymmetries starting from the D9 superbrane. We do both a bottom-up and top-down calculation and compare the two approaches. In section 3 we perform a similar calculation to obtain the maximal $\mathcal{N} = 4 + \mathcal{N} = 4$ DBI action as it follows from the D3 superbrane. The vector branes, that are relevant for a brane interpretation of the half-supersymmetric case, are discussed in section 4. In sections 5 and 6 we present the half-supersymmetric DBI theories in $d = 6$ and $d = 4$ as they follow from the V5 brane and V3 brane, respectively. Our conclusions are given in section 7. We have added three appendices. Appendix A contains the details of the calculation that yields the maximal supersymmetric DBI action while Appendix B contains a similar calculation that leads to the half-supersymmetric DBI action. Finally, some details on our notation are given in Appendix C.

2 DBI-VA with Maximal 16+16 Supersymmetries in $d = 10$ from the D9 superbrane

2.1 Bottom-Up From Supersymmetric Maxwell to Born-Infeld

Our starting point is $\mathcal{N} = 1$, $d = 10$ on-shell Maxwell described above in (1.1). It can be shown that requiring ϵ -supersymmetry up to order $\mathcal{O}(\alpha^2)$ for the first-order non-derivative corrections of the

on-shell multiplet leads to the Born-Infeld combination [4]: ²

$$\begin{aligned}
S = & \int d^{10}x \left\{ -\frac{1}{4}F^2 + \bar{\lambda}\not{\partial}\lambda \right\} - 2\alpha c_4 F^{\mu\nu} \bar{\lambda} \Gamma_\mu \partial_\nu \lambda \\
& + \frac{1}{8}\alpha^2 \left[\text{Tr } F^4 - \frac{1}{4} (F^2)^2 + 4(1 + 4c_4^2) (F^2)^{\mu\nu} \bar{\lambda} \Gamma_\mu \partial_\nu \lambda \right. \\
& + (1 - 4c_4^2) F_\mu^\lambda (\partial_\lambda F_{\nu\rho}) \bar{\lambda} \Gamma^{\mu\nu\rho} \lambda + \frac{1}{2}(c_1 + 8c_4^2) F^2 \bar{\lambda} \not{\partial} \lambda \\
& \left. - \frac{1}{2}c_2 F_{\mu\nu} (\partial_\lambda F^\lambda_\rho) \bar{\lambda} \Gamma^{\mu\nu\rho} \lambda - \frac{1}{2}(c_3 + 4c_4^2) F_{\mu\nu} F_{\rho\sigma} \bar{\lambda} \Gamma^{\mu\nu\rho\sigma} \not{\partial} \lambda \right] \\
& + \mathcal{O}(\alpha^2 \lambda^4) + \mathcal{O}(\alpha^3).
\end{aligned} \tag{2.1}$$

See Appendix C for our notations. The parameters c_1, c_2, c_3 and c_4 cannot be determined. They are related to the redefinitions

$$\begin{aligned}
A_\mu(0) &= A_\mu - \frac{1}{16}\alpha^2 c_2 F^{\nu\rho} \bar{\lambda} \Gamma_{\mu\nu\rho} \lambda, \\
\lambda(0) &= \lambda + \frac{1}{2}\alpha c_4 F_{\mu\nu} \Gamma^{\mu\nu} \lambda + \frac{1}{32}\alpha^2 c_1 F^2 \lambda - \frac{1}{32}\alpha^2 c_3 F_{\mu\nu} F_{\rho\sigma} \Gamma^{\mu\nu\rho\sigma} \lambda,
\end{aligned} \tag{2.2}$$

where $A_\mu(0)$ and $\lambda(0)$ are the fields for $c_i = 0$.

The action (2.1) is invariant up to order α^2 under the following supersymmetry transformations

$$\begin{aligned}
\delta_\epsilon A_\mu &= \bar{\epsilon} \Gamma_\mu \lambda + \frac{1}{2}\alpha c_4 F^{\rho\sigma} \bar{\epsilon} \Gamma_\mu \Gamma_{\rho\sigma} \lambda \\
&+ \frac{1}{32}\alpha^2 (c_1 + 2c_2 - 6) F^2 \bar{\epsilon} \Gamma_\mu \lambda + \frac{1}{8}\alpha^2 (c_2 - 4) (F^2)_\mu^\nu \bar{\epsilon} \Gamma_\nu \lambda \\
&- \frac{1}{16}\alpha^2 (-c_2 + 2c_3 + 2) F_{\mu\nu} F_{\rho\sigma} \bar{\epsilon} \Gamma^{\nu\rho\sigma} \lambda \\
&- \frac{1}{32}\alpha^2 (c_2 + c_3 - 1) F^{\rho\sigma} F^{\lambda\tau} \bar{\epsilon} \Gamma_{\mu\rho\sigma\lambda\tau} \lambda \\
&+ \mathcal{O}(\alpha^2 \lambda^3) + \mathcal{O}(\alpha^3).
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
\delta_\epsilon \lambda &= \frac{1}{4}\Gamma^{\mu\nu} F_{\mu\nu} \epsilon - \frac{1}{8}\alpha c_4 \Gamma^{\mu\nu\rho\sigma} \epsilon F_{\mu\nu} F_{\rho\sigma} + \frac{1}{4}\alpha c_4 F^2 \epsilon \\
&- \frac{1}{128}\alpha^2 (c_1 + 4c_3 + 48c_4^2 - 2) F^2 \Gamma^{ab} F_{ab} \epsilon \\
&- \frac{1}{16}\alpha^2 (c_3 + 8c_4^2 - 1) (F^3)_{\mu\nu} \Gamma^{\mu\nu} \epsilon \\
&+ \frac{1}{384}\alpha^2 (3c_3 + 24c_4^2 + 1) \Gamma^{\mu\nu\rho\sigma\lambda\tau} \epsilon F_{\mu\nu} F_{\rho\sigma} F_{\lambda\tau} \\
&- \alpha c_4 \Gamma^{\mu\nu} \lambda \bar{\epsilon} \Gamma_\nu \partial_\mu \lambda \\
&+ \mathcal{O}(\alpha^2 \lambda^2) + \mathcal{O}(\alpha^3).
\end{aligned} \tag{2.4}$$

We note that there is no choice of redefinition parameters c_1, c_2, c_3, c_4 possible such that the transformation rule of A_μ does not receive any α^2 -corrections. The order α terms are all related to the lowest order using the redefinition with c_4 in (2.2).

²With respect to [4] we have redefined $\alpha^2 \rightarrow -\alpha^2/2$.

The above result is valid in all dimensions where the Majorana flip relations are as in (C.4). For the higher fermion terms one needs also the cyclic identity (A.6), restricting them to $d = 2, 3, 4, 6$ and 10. Remark that when we choose

$$c_1 = -2, \quad c_2 = 4, \quad c_3 = 1, \quad c_4 = 0. \quad (2.5)$$

the transformation laws are not changed with respect to the lowest order ones in $d = 4$.

It appears difficult to continue to higher orders of deformation since one has to find simultaneously the new terms in the supersymmetry deformation and the new terms in the action deformation.

2.2 A Complete Maximal Supersymmetric $d = 10$ DBI-VA Model from the D9 super-brane

The DBI model with complete set of 16+16 global supersymmetries is defined by the gauge-fixed D9-brane action [35–38]

$$S = -\frac{1}{\alpha^2} \int d^{10}x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} - 1 \right\}, \quad (2.6)$$

where³

$$\begin{aligned} G_{\mu\nu} &= \eta_{mn} \Pi_\mu^m \Pi_\nu^n, & \Pi_\mu^m &= \delta_\mu^m - \alpha^2 \bar{\lambda} \Gamma^m \partial_\mu \lambda, & \mu &= 0, 1, \dots, 9, & m &= 0, 1, \dots, 9, \\ \mathcal{F}_{\mu\nu} &\equiv F_{\mu\nu} - b_{\mu\nu}, & b_{\mu\nu} &= -\alpha \bar{\lambda} \Gamma_m \partial_\mu \lambda \Pi_\nu^m - (\mu \leftrightarrow \nu) = -2\alpha \bar{\lambda} \Gamma_{[\nu} \partial_{\mu]} \lambda. \end{aligned} \quad (2.7)$$

The DBI action (2.6) has 16+16 global supersymmetry transformations, which are given in eqs. (89)–(92) in [36] where the term in the transformations that is non-linear in the vectors is somewhat complicated. Here we present an explicit and relatively simple form of all these supersymmetries, based also on [35, 37, 38]. The detailed derivation starting from the κ -symmetric Dp-brane actions is presented in Appendix A. In short, we find from (A.29) and (A.30) the following 16+16 supersymmetries:

Sixteen ϵ transformations, deformation of the Maxwell supermultiplet supersymmetries

$$\begin{aligned} \delta_\epsilon \lambda &= -\frac{1}{2\alpha} (\mathbb{1} - \beta) \epsilon + \xi_\epsilon^\mu \partial_\mu \lambda, \\ \delta_\epsilon A_\mu &= -\frac{1}{2} \bar{\lambda} \Gamma_\mu (\mathbb{1} + \beta) \epsilon + \frac{1}{2} \alpha^2 \bar{\lambda} \Gamma_m (\frac{1}{3} \mathbb{1} + \beta) \epsilon \bar{\lambda} \Gamma^m \partial_\mu \lambda + \xi_\epsilon^\rho F_{\rho\mu} \\ &= \alpha^{-1} \xi_{\epsilon\mu} + \xi_\epsilon^\rho \mathcal{F}_{\rho\mu} - \alpha \xi_\epsilon^\rho \bar{\lambda} \Gamma_\mu \partial_\rho \lambda + \frac{1}{3} \alpha^2 \bar{\epsilon} \Gamma^m \lambda \bar{\lambda} \Gamma_m \partial^\mu \lambda, \end{aligned} \quad (2.8)$$

³Here we use the fact that the terms quartic in λ in $b_{\mu\nu}$ vanish under the $\mu \leftrightarrow \nu$ anti-symmetrization, which allows a nice covariant expression for the 2-form.

Sixteen VA-type ζ transformations

$$\begin{aligned}\delta_\zeta \lambda &= \alpha^{-1} \zeta + \xi_\zeta^\mu \partial_\mu \lambda, \\ \delta_\zeta A_\mu &= \alpha^{-1} \xi_{\zeta\mu} + \xi_\zeta^\rho F_{\rho\mu} - \frac{1}{3} \alpha^2 \bar{\lambda} \Gamma_m \zeta \bar{\lambda} \Gamma^m \partial_\mu \lambda.\end{aligned}\tag{2.9}$$

where

$$\xi_\epsilon^\mu \equiv -\frac{1}{2} \alpha \bar{\lambda} \Gamma^\mu (1 + \beta) \epsilon, \quad \xi_\zeta^\mu = \alpha \bar{\lambda} \Gamma^\mu \zeta,\tag{2.10}$$

and

$$\beta = \mathcal{G} \sum_{k=0}^5 \frac{\alpha^k}{2^k k!} \hat{\Gamma}^{\mu_1 \nu_1 \dots \mu_k \nu_k} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_k \nu_k},\tag{2.11}$$

where \mathcal{G} is defined in (A.14). The expressions (2.10) are the quantities obtained in (A.27).

2.3 Comparing Bottom-Up with Top-Down

To obtain agreement between the step by step deformation of the Maxwell theory with the complete Born-Infeld supersymmetric model following from the brane analysis we must choose our redefinition parameters in the bottom up approach such that all bilinear fermion terms in the action that have a $\gamma^{(3)}$ or higher-gamma structure vanish. Furthermore, by comparing the supersymmetry rules with the brane answer, we deduce that the $(F^3)_{\mu\nu} \gamma^{\mu\nu}$ structure in $\delta\lambda$ should be absent. Fitting the other structures that follow from the brane analysis we find that there is a unique solution for the redefinition parameters that gives agreement with the brane analysis. This choice is given by

$$c_1 = 2, \quad c_2 = 0, \quad c_3 = -1, \quad c_4 = -\frac{1}{2}.\tag{2.12}$$

In this parametrization, the action is given by

$$\begin{aligned}S &= \int d^{10}x \left\{ -\frac{1}{4} F^2 + \bar{\lambda} \not{\partial} \lambda + \alpha F^{\mu\nu} \bar{\lambda} \Gamma_\mu \partial_\nu \lambda \right. \\ &\quad \left. + \frac{1}{8} \alpha^2 \left[\text{Tr } F^4 - \frac{1}{4} (\text{Tr } F^2)^2 + 8 (F^2)^{\mu\nu} \bar{\lambda} \Gamma_\mu \partial_\nu \lambda + 2 F^2 \bar{\lambda} \not{\partial} \lambda \right] \right\} \\ &\quad + \mathcal{O}(\alpha^2 \lambda^4) + \mathcal{O}(\alpha^3).\end{aligned}\tag{2.13}$$

Comparing this result with the brane analysis we may also deduce what the next order corrections to the hidden ζ -supersymmetry are. We start with another basis of the transformations, using the shift symmetry (1.3). The parameter η in (1.3) refers to another basis of the transformations. When we use as independent parameters ϵ and η , the ζ transformations are given by

$$\delta_\zeta = \delta_\epsilon + \delta_\eta, \quad \text{with} \quad \epsilon = -\zeta, \quad \eta = -2\zeta.\tag{2.14}$$

The η transformations are for arbitrary coefficients c_i given by

$$\begin{aligned}
\delta_\eta A^\mu &= \frac{\alpha}{4} \bar{\eta} F^{\nu\mu} \Gamma_\nu \lambda + \frac{\alpha}{8} \bar{\eta} \Gamma^{\mu\nu\rho} F_{\nu\rho} \lambda - \frac{1}{16} \alpha c_2 F_{\nu\rho} \bar{\eta} \Gamma^{\mu\nu\rho} \lambda + \mathcal{O}(\alpha\eta\lambda^3) + \mathcal{O}(\alpha^2), \\
\delta_\eta \lambda &= -\frac{1}{2\alpha} \eta + \alpha \left[-\frac{1}{32} F^2 - \frac{1}{64} \Gamma^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right] \eta \\
&\quad + \frac{1}{4} c_4 F_{\mu\nu} \Gamma^{\mu\nu} \left[\eta - \frac{1}{2} \alpha c_4 F_{\rho\sigma} \Gamma^{\rho\sigma} \eta \right] \\
&\quad + \frac{1}{64} \alpha c_1 F^2 \eta - \frac{1}{64} \alpha c_3 F_{\mu\nu} F_{\rho\sigma} \Gamma^{\mu\nu\rho\sigma} \eta + \mathcal{O}(\alpha\eta\lambda^2) + \mathcal{O}(\alpha^2)
\end{aligned} \tag{2.15}$$

When using the parameters of (2.12) the ϵ transformations simplify to

$$\begin{aligned}
\delta_\epsilon A_\mu &= \bar{\epsilon} \Gamma_\mu \lambda - \frac{1}{4} \alpha \bar{\epsilon} \Gamma_\mu \Gamma \cdot F \lambda \\
&\quad - \frac{1}{8} \alpha^2 F^2 \bar{\epsilon} \Gamma_\mu \lambda - \frac{1}{2} \alpha^2 (F^2)_\mu{}^\nu \bar{\epsilon} \Gamma_\nu \lambda + \frac{1}{16} \alpha^2 F^{\rho\sigma} F^{\lambda\tau} \bar{\epsilon} \Gamma_{\mu\rho\sigma\lambda\tau} \lambda \\
&\quad + \mathcal{O}(\alpha^2\lambda^3) + \mathcal{O}(\alpha^3), \\
\delta_\epsilon \lambda &= \frac{1}{4} \Gamma^{\mu\nu} F_{\mu\nu} \epsilon + \frac{1}{16} \alpha \Gamma^{\mu\nu\rho\sigma} \epsilon F_{\mu\nu} F_{\rho\sigma} - \frac{1}{8} \alpha F^2 \epsilon \\
&\quad - \frac{1}{16} \alpha^2 F^2 \Gamma^{ab} F_{ab} \epsilon + \frac{1}{96} \alpha^2 \Gamma^{\mu\nu\rho\sigma\lambda\tau} \epsilon F_{\mu\nu} F_{\rho\sigma} F_{\lambda\tau} \\
&\quad + \frac{1}{2} \alpha \Gamma^{\mu\nu} \lambda \bar{\epsilon} \Gamma_\nu \partial_\mu \lambda \\
&\quad + \mathcal{O}(\alpha^2\lambda^2) + \mathcal{O}(\alpha^3).
\end{aligned} \tag{2.16}$$

The η transformations are then

$$\begin{aligned}
\delta_\eta A_\mu &= \alpha \frac{1}{8} \bar{\eta} \Gamma \cdot F \Gamma_\mu \lambda + \mathcal{O}(\alpha\eta\lambda^3) + \mathcal{O}(\alpha^2), \\
\delta_\eta \lambda &= -\frac{1}{2\alpha} \eta - \frac{1}{8} \Gamma \cdot F \left[\eta + \frac{1}{4} \alpha F \cdot \Gamma \eta \right] + \mathcal{O}(\alpha\eta\lambda^2) + \mathcal{O}(\alpha^2)
\end{aligned} \tag{2.17}$$

Using (2.14) one can verify that

$$\begin{aligned}
\delta_\zeta A_\mu &= a_\mu(\zeta) + \alpha a^\rho(\zeta) F_{\rho\mu} + \mathcal{O}(\alpha\eta\lambda^3) + \mathcal{O}(\alpha^2), \\
\delta_\zeta \lambda &= \alpha^{-1} \zeta + \alpha a^\mu(\zeta) \partial_\mu \lambda, \\
a^\mu(\zeta) &\equiv \bar{\lambda} \Gamma^\mu \zeta = -\bar{\zeta} \Gamma^\mu \lambda.
\end{aligned} \tag{2.18}$$

To expand the complete action (2.6) in orders of α we use the Mercator formula

$$\begin{aligned}
\det(\mathbb{1} + A) &= \sum_{k=0}^{\infty} \frac{1}{k!} \left(- \sum_{j=1}^{\infty} \frac{(-1)^j}{j} \text{Tr}(A^j) \right)^k \\
&= 1 + \text{Tr} A - \frac{1}{2} \text{Tr} A^2 + \frac{1}{2} (\text{Tr} A)^2 + \frac{1}{3} \text{Tr} A^3 - \frac{1}{2} (\text{Tr} A)(\text{Tr} A^2) + \frac{1}{6} (\text{Tr} A)^3 + \mathcal{O}(A^4)
\end{aligned} \tag{2.19}$$

where A is an arbitrary dimensionless $n \times n$ matrix. To apply this to (2.7) we use

$$\begin{aligned}
A_\mu{}^\nu &= \alpha \mathcal{F}_\mu{}^\nu - \alpha^2 \bar{\lambda} \Gamma_\mu \partial^\nu \lambda - \alpha^2 \bar{\lambda} \Gamma^\nu \partial_\mu \lambda + \alpha^4 (\bar{\lambda} \Gamma^m \partial_\mu \lambda) (\bar{\lambda} \Gamma^m \partial^\nu \lambda) \\
&= \alpha F_\mu{}^\nu - 2\alpha^2 \bar{\lambda} \Gamma_\mu \partial^\nu \lambda + \alpha^4 (\bar{\lambda} \Gamma^m \partial_\mu \lambda) (\bar{\lambda} \Gamma^m \partial^\nu \lambda).
\end{aligned} \tag{2.20}$$

The result agrees with (2.13) and gives us moreover the terms $\mathcal{O}(\alpha^2\lambda^4)$:

$$\begin{aligned}
S = & \int d^{10}x \left\{ -\frac{1}{4}(F^2) + \bar{\lambda}\not{\partial}\lambda - \alpha F_\mu{}^\rho \bar{\lambda}\Gamma_\rho \partial^\mu \lambda \right. \\
& + \alpha^2 \left[\frac{1}{8}(\text{Tr } F^4) - \frac{1}{32}(F^2)^2 + (F^2)_\mu{}^\rho \bar{\lambda}\Gamma_\rho \partial^\mu \lambda + \frac{1}{4}\bar{\lambda}\not{\partial}\lambda(F^2) \right. \\
& \left. \left. - \frac{1}{2}(\bar{\lambda}\not{\partial}\lambda)^2 - \frac{1}{2}(\bar{\lambda}\Gamma^m \partial_\mu \lambda)(\bar{\lambda}\Gamma^m \partial^\mu \lambda) + \bar{\lambda}\Gamma_\mu \partial^\rho \lambda \bar{\lambda}\Gamma_\rho \partial^\mu \lambda \right] + \mathcal{O}(\alpha^3) \right\}
\end{aligned} \tag{2.21}$$

An easy check of the new terms is provided by calculating the variation of the action under ζ symmetry proportional to $\alpha\lambda^3$. These terms are not influenced by the redefinitions (2.2), and thus these can also be inserted in the general expression (2.1), and thus also in (2.13). Other reparametrizations of the type $\lambda \rightarrow c_5\alpha^2\lambda^2\partial\lambda$ do modify these terms.

The ζ supersymmetry rules (2.9) can be easily expanded and agree with (2.18). To compare the ϵ transformation rules, we need the following expansion of (2.11):

$$\begin{aligned}
\beta = & \mathcal{G} \left[1 + \frac{1}{2}\alpha\Gamma \cdot \mathcal{F} + \frac{1}{8}\alpha^2\Gamma^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \right. \\
& + \frac{1}{2}\alpha^3\Gamma^{\mu\nu\rho\sigma}F_{\mu\nu}(\bar{\lambda}\Gamma_\sigma\partial_\rho\lambda) + \frac{1}{48}\alpha^3\Gamma^{\mu\nu\rho\sigma\lambda\tau}F_{\mu\nu}F_{\rho\sigma}F_{\lambda\tau} + \alpha^3\Gamma^{\mu\rho}(\bar{\lambda}\Gamma^\nu\partial_\rho\lambda)F_{\mu\nu} \left. \right] \\
& + \mathcal{O}(\alpha^4), \\
\mathcal{G} = & 1 - \frac{1}{4}\alpha^2F^2 + \alpha^3F_{\mu\nu}\bar{\lambda}\Gamma^\mu\partial^\nu\lambda + \mathcal{O}(\alpha^4).
\end{aligned} \tag{2.22}$$

The first expression was again obtained using (2.19), where now

$$\begin{aligned}
A_\mu{}^\nu = & \alpha\mathcal{F}_{\mu\rho}G^{\rho\nu} = \alpha(F_{\mu\rho} + 2\alpha\bar{\lambda}\Gamma_{[\rho}\partial_{\mu]}\lambda) \left(\eta^{\rho\nu} + 2\alpha^2\bar{\lambda}\Gamma^{(\rho}\partial^{\nu)}\lambda + \mathcal{O}(\alpha^4) \right) \\
= & \alpha F_\mu{}^\nu + \alpha^2\bar{\lambda}\Gamma^\nu\partial_\mu\lambda - \alpha^2\bar{\lambda}\Gamma_\mu\partial^\nu\lambda + 2\alpha^3F_{\mu\rho}\bar{\lambda}\Gamma^{(\rho}\partial^{\nu)}\lambda + \mathcal{O}(\alpha^4),
\end{aligned} \tag{2.23}$$

such that traces of odd powers of A vanish. In this way one obtains

$$\begin{aligned}
\delta_\epsilon A_\mu = & -\bar{\lambda}\Gamma_\mu\epsilon - \frac{1}{4}\alpha\bar{\epsilon}\Gamma_\mu\Gamma \cdot \mathcal{F}\lambda \\
& - \frac{1}{8}\alpha^2F^2\bar{\epsilon}\Gamma_\mu\lambda - \frac{1}{2}\alpha^2(F^2)_\mu{}^\nu\bar{\epsilon}\Gamma_\nu\lambda \\
& + \frac{1}{16}\alpha^2F^{\rho\sigma}F^{\lambda\tau}\bar{\epsilon}\Gamma_{\mu\rho\sigma\lambda\tau}\lambda \\
& + \alpha^2\bar{\lambda}\Gamma^\rho\epsilon\bar{\lambda}(\Gamma_\mu\partial_\rho\lambda - \frac{1}{3}\Gamma_\rho\partial_\mu\lambda) + \mathcal{O}(\alpha^3).
\end{aligned} \tag{2.24}$$

This agrees with the bottom up results, adding the last line, and the term implicit in the order α term with \mathcal{F} , as $\mathcal{O}(\alpha^2\lambda^3)$ terms. For the ϵ transformation of λ we find

$$\begin{aligned}
\delta_\epsilon \lambda = & \frac{1}{4}\hat{\Gamma} \cdot \mathcal{F}\epsilon - \frac{1}{8}\alpha\epsilon\mathcal{F}^2 + \frac{1}{16}\alpha\Gamma^{\mu\nu\rho\sigma}\epsilon\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma} \\
& - \frac{1}{16}\alpha^2F^2\Gamma^{ab}F_{ab}\epsilon + \frac{1}{96}\alpha^2\Gamma^{\mu\nu\rho\sigma\lambda\tau}\epsilon F_{\mu\nu}F_{\rho\sigma}F_{\lambda\tau} \\
& - \alpha\partial_\mu\lambda\bar{\lambda}\Gamma^\mu\epsilon - \frac{1}{4}\alpha^2\partial_\mu\lambda\bar{\lambda}\Gamma^\mu\Gamma \cdot F\epsilon + \mathcal{O}(\alpha^3).
\end{aligned} \tag{2.25}$$

Thus, there are higher order fermions included in the first term in

$$\hat{\Gamma}^{\mu\nu} = \Gamma^{\mu\nu} - 2\alpha^2 \Gamma^{\rho[\mu} (\bar{\lambda} \Gamma^{\nu]} \partial_\rho \lambda) + \mathcal{O}(\alpha^4), \quad (2.26)$$

in \mathcal{F} , and in the last term in (2.25). We have agreement with (2.16) apart from the order α cubic fermion terms. It can be shown (after using the Fierz identity (A.6)) that the difference is proportional to a field equation and is thus a ‘zilch symmetry’.⁴

2.4 16+16 Supersymmetry Algebra in $d = 10$

The algebra of the ϵ and ζ supersymmetries is

$$\begin{aligned} [\delta(\epsilon_1), \delta(\epsilon_2)] &= \delta_{cP}(\xi_{\epsilon\epsilon}^\mu) + \text{field equations} + \mathcal{O}(\alpha^2), \quad \xi_{\epsilon\epsilon}^\mu = \bar{\epsilon}_1 \Gamma^\mu \epsilon_2, \\ [\delta(\epsilon), \delta(\zeta)] &= \delta_{cP}(\xi_{\epsilon\zeta}^\mu) + \delta_{U(1)}(\Lambda_{\epsilon\zeta}), \quad \xi_{\epsilon\zeta}^\mu = \bar{\zeta} \Gamma^\mu \epsilon, \\ \Lambda_{\epsilon\zeta} &= x_\mu \alpha^{-1} \xi_{\epsilon\zeta}^\mu + \frac{1}{3} \xi_\zeta^m \bar{\lambda} \Gamma_m \beta \epsilon + \xi_\zeta^\rho \xi_\epsilon^\sigma (F_{\rho\sigma} - \frac{1}{3} \alpha \bar{\lambda} \Gamma_\rho \partial_\sigma \lambda + \alpha \bar{\lambda} \Gamma_\sigma \partial_\rho \lambda) \\ &\quad + \frac{1}{3} \alpha^2 \xi_\zeta^\rho (\bar{\lambda} \Gamma^m \epsilon) (\bar{\lambda} \Gamma_m \partial_\rho \lambda), \\ [\delta(\zeta_1), \delta(\zeta_2)] &= \delta_{cP}(\xi_{\zeta\zeta}^\mu) + \delta_{U(1)}(\Lambda_{\zeta\zeta}), \quad \xi_{\zeta\zeta}^\mu = 2 \bar{\zeta}_1 \Gamma^\mu \zeta_2, \\ \Lambda_{\zeta\zeta} &= \frac{1}{\alpha} \xi_{\zeta\zeta}^\mu x_\mu + \xi_{\zeta 2}^\rho \xi_{\zeta 1}^\sigma (F_{\rho\sigma} - \frac{2}{3} \alpha \bar{\lambda} \Gamma_{[\rho} \partial_{\sigma]} \lambda), \end{aligned} \quad (2.27)$$

where ξ_ϵ and ξ_ζ are the expressions in (2.10). Here δ_{cP} is a covariant translation, i.e. a spacetime translation combined with an Abelian gauge transformation $\delta A_\mu = \partial_\mu \Lambda$ with parameter $\Lambda = -\xi^\mu A_\mu$. The first commutator is only valid on-shell. When using to this order the transformations of λ as obtained in (2.16) the commutator closes without using field equations on A_μ , and using field equations on the fermion field. However, with the transformations as in (2.25), the field equations are also needed for the commutator on the gauge field.

Note that the first terms of $\Lambda_{\epsilon\zeta}$ and $\Lambda_{\zeta\zeta}$ (proportional to x^μ) can also be understood as shift transformations of the vector field.

3 DBI-VA with Maximal 16 + 16 Supersymmetry in $d = 4$ from the D3 superbrane

The complete DBI action in $d = 4$ is relatively simple when the fermion part of the action is “packaged into a $d = 10$ form”, namely all spinors are still $d = 10$ Majorana-Weyl spinors and all $\Gamma^{m'}$ and Γ^I are $d = 10$ matrices. We find

$$S = -\frac{1}{\alpha^2} \int d^4x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} - 1 \right\}, \quad \mu = 0, 1, 2, 3, \quad (3.1)$$

⁴Note that also in the comparison between different formulations of the VA actions [32, 40, 41] in [42], such symmetries, also called ‘trivial symmetries’, were involved.

where

$$\begin{aligned}
G_{\mu\nu} &= \eta_{mn}\Pi_\mu^m\Pi_\nu^n = \eta_{m'n'}\Pi_\mu^{m'}\Pi_\nu^{n'} + \delta_{IJ}\Pi_\mu^I\Pi_\nu^J, \quad m' = 0, 1, 2, 3, \quad I = 1, \dots, 6, \\
\Pi_\mu^{m'} &= \delta_\mu^{m'} - \alpha^2 \bar{\lambda} \Gamma^{m'} \partial_\mu \lambda, \quad \Pi_\mu^I = \partial_\mu \phi^I - \alpha^2 \bar{\lambda} \Gamma^I \partial_\mu \lambda, \quad \mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - b_{\mu\nu}, \\
b_{\mu\nu} &= 2\alpha \bar{\lambda} \Gamma_{[\mu} \partial_{\nu]} \lambda - 2\alpha \bar{\lambda} \Gamma_I \partial_{[\mu} \lambda \partial_{\nu]} \phi^I = -2\alpha \bar{\lambda} \Gamma_{m'} \partial_{[\mu} \lambda \Pi_{\nu]}^{m'} - 2\alpha \bar{\lambda} \Gamma_I \partial_{[\mu} \lambda \Pi_{\nu]}^I. \quad (3.2)
\end{aligned}$$

This action has a maximal number of supersymmetries. The 16 ϵ -supersymmetries correspond to a deformation of the original 16 supersymmetries of the $\mathcal{N} = 4$, $d = 4$ Maxwell multiplet, while the 16 ζ -supersymmetries correspond to VA-type supersymmetries. Explicitly, we find the following transformation rules:

Sixteen ϵ transformations, deformation of the Maxwell supermultiplet

$$\begin{aligned}
\delta_\epsilon \phi^I &= \frac{1}{2} \alpha \bar{\lambda} \Gamma^I [\mathbb{1} + \beta] \epsilon + \xi_\epsilon^\mu \partial_\mu \phi^I, \\
\delta_\epsilon \lambda &= -\frac{1}{2\alpha} [\mathbb{1} - \beta] \epsilon + \xi_\epsilon^\mu \partial_\mu \lambda, \\
\delta_\epsilon A_\mu &= -\frac{1}{2} \bar{\lambda} (\Gamma_\mu + \Gamma_I \partial_\mu \phi^I) [\mathbb{1} + \beta] \epsilon \\
&\quad + \frac{1}{2} \alpha^2 \bar{\lambda} \Gamma_m [\frac{1}{3} \mathbb{1} + \beta] \epsilon \bar{\lambda} \Gamma^m \partial_\mu \lambda + \xi_\epsilon^\rho F_{\rho\mu}. \quad (3.3)
\end{aligned}$$

Sixteen VA-type ζ supersymmetry transformations

$$\begin{aligned}
\delta_\zeta \phi^I &= -\alpha \bar{\lambda} \Gamma^I \zeta + \xi_\zeta^\mu \partial_\mu \phi^I, \\
\delta_\zeta \lambda &= \alpha^{-1} \zeta + \xi_\zeta^\mu \partial_\mu \lambda, \\
\delta_\zeta A_\mu &= \bar{\lambda} (\Gamma_\mu + \Gamma_I \partial_\mu \phi^I) \zeta + \xi_\zeta^\rho F_{\rho\mu} - \frac{1}{3} \alpha^2 \bar{\lambda} \Gamma_m \zeta \bar{\lambda} \Gamma^m \partial_\mu \lambda, \quad (3.4)
\end{aligned}$$

where ξ_ϵ^μ and ξ_ζ^μ are given in (2.10),

$$\begin{aligned}
\beta &= -i\mathcal{G} \sum_{k=0}^2 \frac{\alpha^k}{2^k k!} \hat{\Gamma}^{\mu_1 \nu_1 \dots \mu_k \nu_k} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_k \nu_k} \Gamma_{(0)}^{D3} \Gamma_*^{(3)} = 1 + \mathcal{O}(\alpha), \\
\Gamma_{(0)}^{D3} &= \frac{1}{4! \sqrt{|G|}} \varepsilon^{\mu_1 \dots \mu_4} \hat{\Gamma}_{\mu_1 \dots \mu_4} = i\Gamma_*^{(3)} + \mathcal{O}(\alpha), \quad \Gamma_*^{(3)} = -i\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3, \quad (3.5)
\end{aligned}$$

and terms with $\partial_\mu \phi^I$ are considered as $\mathcal{O}(\alpha)$.

The action also has a shift symmetry

$$\delta \phi^I = a^I. \quad (3.6)$$

The scalars in the $d = 4$ action (3.1) originate from the 6 directions X^I transverse to the D3-brane, see Appendix A. This symmetry is a surviving part of the Poincaré translation symmetry in $d = 10$, $X^I \rightarrow X^I + a^I$ for $I = 1, \dots, 6$.

3.1 16+16 Supersymmetry Algebra in $d = 4$

One can use $d = 10$ algebra via dimensional reduction to $d = 4$ or work out the explicit $d = 4$ algebra directly. Here we are mostly interested in the way the second supersymmetry and the combination of the first and second acts on the scalars.

$$\begin{aligned} [\delta(\zeta_1), \delta(\zeta_2)]\phi^I &= 2\bar{\zeta}_2(\Gamma^I - \Gamma^\mu \partial_\mu \phi^I)\zeta_1, \\ [\delta(\epsilon), \delta(\zeta)]\phi^I &= \bar{\zeta}(\Gamma^\mu \partial_\mu \phi^I - \Gamma^I)\epsilon. \end{aligned} \quad (3.7)$$

A shift symmetry on scalars is an important feature of duality symmetries: in particular, in the case of $E_{7(7)}$ symmetry in $N = 8$ supergravity the single-soft scalar limits were studied in detail in [43] and used to prove the UV finiteness below 7 loop order. Thus we see from (3.7) that the shift of scalars that is a symmetry in the action appears already in the algebra of the extra supersymmetries.

3.2 Maxwell $d = 4$ Supermultiplet with $SU(4)$ symmetry

We would like to rewrite the fermionic sector of the action (3.1) and its symmetries using the four $d = 4$ Majorana fermions and the $d = 4$ γ -matrices, following [44, 45] where the dimensional reduction from 10 to 4 was performed in SYM theory, and the $SU(4)$ symmetry of the model was revealed. Moreover, we would like to bring the complete action to the form which at the quadratic level coincides with the one in [1, 46] where we have a $d = 4$, $\mathcal{N} = 4$ Maxwell multiplet. We would like to deform the $\mathcal{N} = 4$ Maxwell action, namely

$$S_{\text{Maxw}} = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2\bar{\psi}_i \not{\partial} \psi^i - \frac{1}{8}\partial_\mu \varphi_{ij} \partial^\mu \varphi^{ij} \right), \quad (3.8)$$

where in [46] the $d = 4$ left-handed chiral spinor is assigned to the fundamental representation of $SU(4)$, and carries an upper $SU(4)$ index; right-handed components will then transform according to the conjugate representation, in agreement with the Majorana property and have a lower index:

$$\psi_i = -\gamma_* \psi^i, \quad \psi^i \equiv (\psi_i)^C = \gamma_* \psi^i, \quad \varphi^{ij} \equiv (\varphi_{ij})^* = -\frac{1}{2}\varepsilon^{ijkl}\varphi_{kl}. \quad (3.9)$$

The action (3.8) is invariant under the 16 supersymmetries

$$\delta_\epsilon A_\mu = \bar{\epsilon}^i \gamma_\mu \psi_i + \bar{\epsilon}_i \gamma_\mu \psi^i, \quad \delta \psi_i = \frac{1}{4}\gamma \cdot F^+ \epsilon_i + \frac{1}{2}\not{\partial} \varphi_{ij} \epsilon^j, \quad \delta \varphi_{ij} = 4\bar{\epsilon}_{[i} \psi_{j]} - 2\varepsilon_{ijkl}\bar{\epsilon}^k \psi^\ell. \quad (3.10)$$

The second, trivial, set of η symmetries is

$$\delta A_\mu = 0, \quad \delta \varphi^{ij} = 0, \quad \delta \psi_i = -\frac{1}{2\alpha}\eta_i, \quad \delta \psi^i = -\frac{1}{2\alpha}\eta^i. \quad (3.11)$$

We can write the φ_{ij} in terms of 6 real components as

$$\varphi_{ij} \equiv \varphi_a \beta_{ij}^a - i\varphi_{a+3} \alpha_{ij}^a, \quad \varphi^{ij} \equiv (\varphi_{ij})^* = -\frac{1}{2}\varepsilon^{ijkl}\varphi_{kl}, \quad (3.12)$$

where $a = 1, 2, 3$ and α, β are the real antisymmetric 4×4 matrices of $SU(2) \times SU(2)$ given explicitly in [45] and whose properties are given in (C.7).

Now we indicate how this can be seen as the $\alpha = 0$ limit of the action (3.1) and the transformation rules (3.3). We therefore define the 6 scalars ϕ^I and $d = 10$ chiral Majorana spinor λ as

$$\phi^I = \alpha \varphi^I, \quad \lambda = \begin{pmatrix} \psi^i \\ \psi_i \end{pmatrix}, \quad \bar{\lambda} = \begin{pmatrix} \bar{\psi}_i & \bar{\psi}^i \end{pmatrix}, \quad (3.13)$$

using the decomposition of the $d = 10$ spinors in $d = 4$ spinors according to the Clifford representation (C.6). Contractions of two $d = 10$ vectors in a $d = 4$ part and the $SO(6)$ part is done according to

$$A^m B_m = A^\mu B^\nu \eta_{\mu\nu} + A^I B^J \delta_{IJ} = A^\mu B^\nu \eta_{\mu\nu} + (A^a B^b \delta_{ab} + A^{a+3} B^{b+3} \delta_{ab}), \quad (3.14)$$

and when using the scalars as in (3.12) this implies that

$$\varphi_a \varphi_b \delta^{ab} + \varphi_{a+3} \varphi_{b+3} \delta^{ab} = \frac{1}{8} \varphi_{ij} \varphi^{ij}. \quad (3.15)$$

Using also for the ϵ parameters the decomposition as in (3.13) (and for convenience using ϵ^i and ϵ_i as components of ϵ) we can write e.g.

$$\begin{aligned} \bar{\epsilon} \Gamma_\mu \lambda &= \bar{\epsilon}_i \gamma_\mu \psi^i + \bar{\epsilon}^i \gamma_\mu \psi_i, \\ \bar{\epsilon} \Gamma^a \lambda &= -\bar{\epsilon}_i \beta^{a\,ij} \psi_j - \bar{\epsilon}^i \beta_{ij}^a \psi^j, \\ \bar{\epsilon} \Gamma^{a+3} \lambda &= -i \bar{\epsilon}_i \alpha^{a\,ij} \psi_j + i \bar{\epsilon}^i \alpha_{ij}^a \psi^j. \end{aligned} \quad (3.16)$$

This allows to check that the $\alpha = 0$ part of the action (3.1) and transformations (3.3) agree with the action (3.8) and transformations (3.10). Also the lowest order (α^{-1}) of (3.4) equals (3.11) using the translation (2.14). Therefore the full action and transformations (3.1) and (3.3) are a deformation of the lowest order $d = 4$, $\mathcal{N} = 4$ theory.

The form of the action and supersymmetries in $SU(4)$, $d = 4$ fermion notation is significantly more complicated than the one above with $d = 10$ packaging of fermions in $d = 4$ action and in supersymmetry rules. One can view this fact as a matter of notational convenience. Using the four four-component Majorana spinors in $d = 4$ leads to an increasing complexity of the complete non-linear action and its supersymmetries. We therefore leave it in the form given in (3.1)-(3.4) with understanding that it codifies all information which may, in principle, be expressed also using the $d = 4$ spinors and $SU(4)$ symmetry of the theory, as in the linearized action in (3.8).

4 Vector Superbranes in $d = 6$ and DBI-VA dynamics

In the previous part of the paper and in the appendix A we have constructed the worldvolume action of Dirichlet branes in a flat background with 32 supersymmetries. These D-branes occur as solutions of

IIA and IIB supergravity. A characteristic feature of these Dirichlet branes is that their worldvolume dynamics is described by a vector supermultiplet. The scalars in this multiplet are the embedding scalars and the vector is the Born-Infeld vector. A special case is the D9-brane which has no embedding scalars at all. We have seen how, starting from a kappa-symmetric worldvolume action of the D9-brane, this has led, after gauge-fixing, to a supersymmetric DBI action with 16+16 supersymmetries. The existence of this D9-brane suggests the existence of all other Dp-branes, with $0 \leq p \leq 8$, by dimensional reduction of the world-volume. More precisely, reducing the $d = 10$ supersymmetric DBI action to $p + 1$ dimensions leads to a supersymmetric DBI action in $p + 1$ dimensions with $d - p - 1$ scalars and one DBI vector. This is precisely the worldvolume content of the Dp-brane.

In Appendix B we describe in a similar way brane actions with vector multiplets and 8+8 supersymmetries. Such branes do not occur in $d = 10$ Heterotic or Type I supergravity. In the case of 9-branes, this is consistent with the fact that vector multiplets with 8+8 supersymmetries only occur in $d \leq 6$ dimensions. It is therefore natural to look for branes with vector multiplets and 8+8 supersymmetries in $d = 6$ half-maximal supergravity. There are three half-maximal $d = 6$ supergravities: Heterotic and Type I, with non-chiral (1,1) supersymmetry, and chiral iib supergravity, with (2,0) supersymmetry. Note that the Heterotic theory is S-dual to Type I and that the Type I theory is T-dual to iib, see, e.g., [47]. The heterotic supergravity contains the (1,1) supergravity with 20 vector multiplets. Apart from the scalars that transform in the $SO(4, 20)$ isometry group, it contains one ‘dilaton’ that is invariant under this group. The iib theory is a (2,0) supergravity coupled to 21 tensor multiplets, whose scalars transform under a $SO(5, 21)$ isometry group. There is no other invariant scalar, and therefore no corresponding string coupling constant.⁵

In searching for branes with a worldvolume vector multiplet, it is important to keep in mind that these branes are not necessarily Dirichlet branes in the sense that their tensions scale with the inverse string coupling constant. In fact, $d = 6$ Heterotic supergravity has no branes whose tension scales with g_s^{-1} , whereas the Type I theory has only such branes with worldvolume hypermultiplets. For our purposes, however, all we need is a brane whose worldvolume dynamics is described by a vector multiplet. We do not mind that the tension of such branes scale differently than the Dirichlet branes. Let us call from now the branes with a worldvolume vector multiplet ‘vector branes’.

We focus here on the space-filling vector 5-branes (called V5-branes) with a 6-dimensional worldvolume and the vector 3-branes (V3-branes) with a 4-dimensional worldvolume. Note that the V5-branes couple to 6-forms, which are not part of the supergravity multiplet that describes physical degrees of freedom. The V3-branes are ‘defect-branes’ with two transverse directions. They couple to 4-forms

⁵Both theories reduce to the $\mathcal{N} = 4$ theory in $d = 5$, where the ‘dilaton’ is present

that are dual to the scalars of the supergravity multiplet. In [39] an analysis was given of the world-volume content of the different branes of $d = 6$ half-maximal supergravity. This analysis was based on the construction of a gauge-invariant Wess-Zumino (WZ) term consistent with worldvolume supersymmetry. The requirement that such a WZ term can be constructed is a necessary condition for a kappa-symmetric worldvolume action to exist. In this work we will only consider branes in a flat background. Based on this WZ analysis it was concluded in [39] that V5-branes occur both in $d = 6$ Heterotic, Type I and iib supergravity whereas V3-branes only occur in $d = 6$ iib supergravity. In all cases the tension of the branes is not proportional to the inverse string coupling constant like it was the case for the tension of the IIA and IIB D-branes. For our purposes, it is sufficient to restrict to the V-branes of $d = 6$ iib supergravity.

Following the analysis of appendix A we construct in Appendix B the supersymmetric DBI actions with 8+8 supersymmetries describing the V_p -branes ($p = 1, 3, 5$) of $d = 6$ iib supergravity. We use these branes to find the half-maximal susy DBI actions.⁶

5 DBI-VA with Half-Maximal 8+8 Supersymmetry in $d = 6$ from the V5 brane

We could have used the D5 brane of IIB $d = 10$ theory to get the DBI in $d = 6$ with maximal supersymmetry and, of course, we could have truncated it to the half of supersymmetry. Alternatively, we may use the vector branes available in $d = 6$ which have been constructed in Appendix B. The $d = 6$ DBI action with 8+8 supersymmetries (using V5 brane) is given by

$$S = -\frac{1}{\alpha^2} \int d^6x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} - 1 \right\}, \quad (5.1)$$

where $\mu = 0, 1, \dots, 5$. The quantities that appear here are the same expressions as in (2.7), where now $m = 0, 1, \dots, 5$. Here λ is a symplectic $d = 6$ Majorana-Weyl spinor and the symplectic indices i in a bilinear are contracted using the antisymmetric ε_{ij} , see (C.5). The above DBI action is invariant under the eight ϵ transformations, deformation of the $d = 6$ Maxwell supersymmetries, and eight VA-type ζ transformations as expressed in (2.8) and (2.9). We use now for Γ^m and Γ_μ the flat $d = 6$ matrices.

$$\beta = \mathcal{G} \sum_{k=0}^3 \frac{\alpha^k}{2^k k!} \hat{\Gamma}^{\mu_1 \nu_1 \dots \mu_k \nu_k} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_k \nu_k} = 1 + \mathcal{O}(\alpha), \quad (5.2)$$

based on the matrices $\hat{\Gamma}_\mu$, which are the pull-back to the word-volume matrix of $d = 6$.

⁶Note that V1-branes are special in the sense that a vector on a two-dimensional worldvolume is equivalent to an integration constant.

6 DBI-VA with Half-Maximal 8 + 8 Supersymmetry in $d = 4$ from the V3 brane

As in the maximal supersymmetry case, the $d = 4$ DBI model with half-maximal supersymmetry is relatively simple when all fermions are packaged in $d = 6$ symplectic Majorana fermions and all the Γ -matrices are the ones in $d = 6$.

$$S = -\frac{1}{\alpha^2} \int d^4x \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} - 1 \right\}, \quad \mu = 0, 1, 2, 3, \quad (6.1)$$

where the definitions (3.2) apply with now $m = 0, \dots, 5$, $m' = 0, \dots, 3$ and $I = 1, 2$. The 8+8 supersymmetries are given by (3.3), (3.4) where, however, the spinors are $d = 6$ symplectic Majorana spinors and the Γ 's are those from $d = 6$.

Finally, we comment on the $d = 4$ DBI action with half-maximal supersymmetry and $SU(2)$ symmetry. If we would have a simple action for $d = 4$ DBI with maximal supersymmetry and $SU(4)$ symmetry, truncating it to half-maximal case with $SU(2)$ symmetry would be extremely simple, we would just allow the $SU(4)$ index take values not in $i = 1, 2, 3, 4$ but in $i = 1, 2$. However, as explained above, the complete action is simple only when fermions are in the higher dimensional form. So, here, the procedure of switching to the DBI action with manifest $SU(2)$ would be the same as in the previous case with manifest $SU(4)$. The expressions for the action and supersymmetries become complicated, but it is clear that in principle all information in the BI action above can be transferred into an $SU(2)$ covariant action.

7 Discussion

We presented here an explicit completion to all orders of the deformation of the Maxwell supermultiplets with maximal supersymmetry in $d = 10, 4$ and half-maximal ones in $d = 6, 4$. The deformation of the global supersymmetry of the Maxwell multiplet to all orders is required. It is also accompanied by a non-linear extra supersymmetry of the Volkov-Akulov type of the same dimension: the maximal case has in total 16+16 supersymmetries and the half-maximal one has in total 8+8 of these supersymmetries. Both our maximal supersymmetry and half-maximal supersymmetry models are realized in the DBI type actions: when all spinors and scalars are absent, we recover the classical BI models, for example in $d = 10$ we find

$$S_{\text{BI}} = -\frac{1}{\alpha^2} \int d^{10}x \left\{ \sqrt{-\det(\eta_{\mu\nu} + \alpha F_{\mu\nu})} - 1 \right\}, \quad (7.1)$$

Also, it is interesting that in $d = 10$, when the covariant 2-form $\mathcal{F}_{\mu\nu}$ is absent, the same action is a $d = 10$ analog, as discussed in [38], of the $d = 4$ Volkov-Akulov action [32] (at $\alpha = 1$ and ignoring the

constant part of the action)

$$S_{\text{VA}} = - \int d^{10}x \sqrt{-\det G_{\mu\nu}} = \int E^{m_0} \wedge \dots \wedge E^{m_9} \quad (7.2)$$

$$E^m = dx^m + \bar{\lambda} \Gamma^m d\lambda. \quad (7.3)$$

The general maximal (half-maximal) supersymmetric Dirac-Born-Infeld-Volkov-Akulov models presented in this paper, are only slightly more complicated than the DBI and VA actions shown above. The Lagrangian is always of the form

$$S_{\text{DBI-VA}} = -\frac{1}{\alpha^2} \left(\sqrt{-\det(G_{\mu\nu} + \alpha \mathcal{F}_{\mu\nu})} - 1 \right) \quad (7.4)$$

where $G_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$ are defined for each case for $d = 10$, $d = 4$ in the maximum and $d = 6$, $d = 4$ in the half-maximum supersymmetry, in the relevant sections of the paper. The all order in deformation supersymmetries are in all cases given by rather complicated expressions, which involves the 2-forms and the fermion dependent pull-back world-volume matrices, related to (7.3). However, the exact hidden non-linear supersymmetry transformation of fermions is a simple shift and quadratic in fermions expression which is literally the original Volkov-Akulov formula

$$\delta_\zeta \lambda = \alpha^{-1} \zeta + \alpha \bar{\lambda} \Gamma^\mu \zeta \partial_\mu \lambda. \quad (7.5)$$

Despite the complicated dependence of supersymmetries on $\mathcal{F}_{\mu\nu}$, the complete models in $d = 10, 6, 4$ are given on one page each, using a notation which we called ‘fermions packaged in a $d = 10$ ($d = 6$) form’ for the maximal (half-maximal) case.

It would be interesting to compare our new models with the known half-maximal models in $d = 4$ [10–13, 15–18, 23–27]. with various amount of supersymmetry. The models with 8+8 supersymmetries are known only up to a certain level of deformation, whereas our model with the same amount of supersymmetries is known to all orders of deformation.⁷ In models studied in [18, 25, 27], 8 supersymmetries are manifest and undeformed, whereas the hidden 8 are deformed. The comparison between these models and our complete model will likely be possible with account of field redefinitions which relate various versions of the VA models even in absence of the vector field. Recently it was shown how various forms of a goldstino action are related to each other by a non-linear local field redefinition [40–42, 49–52].

The DBI models with 16+16 supersymmetries do not seem to be given in the literature. Part of the reason for the absence of such models is that in case of 8+8 supersymmetries, the $\mathcal{N} = 2$, $d = 4$

⁷The existence of the complete 8+8 DBI-VA model in $d = 4$ described in this paper helps to explain certain puzzles concerning hidden supersymmetry and duality of some 4-dimensional models with manifest $\mathcal{N} = 2$ supersymmetry and hidden $\mathcal{N} = 2$ supersymmetry [48].

superfields were used. However for $\mathcal{N} = 4$ no off shell superfields are available, therefore the tools used for 8+8 models are not available for 16+16. Meanwhile we have shown that using the superbrane approach it is not necessary to keep any linear supersymmetries manifest. The theory includes a natural Born-Infeld non-supersymmetric model and, in absence of 2-form $\mathcal{F}_{\mu\nu}$, has natural Goldstino variables in which the action has an original Volkov-Akulov form.

Finally, we would like to discuss our original goal to find new ways to construct supersymmetric invariants for theories with extended supersymmetries where there are no known auxiliary fields. We have learned here how to build the deformation of the $\mathcal{N} = 4$, $d = 4$ Maxwell multiplet resulting in a model with $\mathcal{N} = 4$ deformed supersymmetry and another $\mathcal{N} = 4$ hidden supersymmetry. However, these supersymmetries are global: they originate from the super-branes extended objects which are known to interact with the supergravity satisfying classical field equations. On shell background supergravity is a condition for the local fermionic κ -symmetry. Some new ideas will be required to build the higher derivative superconformal invariants, if they exist, to shed some light on the issue of the UV finiteness of extended perturbative supergravity, as proposed in [1].

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A Appendix: Dp-branes

Here we describe Dp superbranes in IIB theory with positive chiral spinors.

A.1 Dp-superbrane with local κ -symmetry, maximal supersymmetry and general covariance

The κ -symmetric Dp-brane action (with $p = 2n + 1$ odd), in a flat background geometry with (longitudinal and transverse) coordinates X^m , $m = 0, \dots, 9$, consists of the Dirac-Born-Infeld-Nambu-Goto term S_{DBI} and Wess-Zumino term S_{WZ} in the world-volume coordinates σ^μ ($\mu = 0, \dots, p$):

$$S_{\text{DBI}} + S_{\text{WZ}} = -\frac{1}{\alpha^2} \int d^{p+1}\sigma \sqrt{-\det(G_{\mu\nu} + \alpha\mathcal{F}_{\mu\nu})} + \frac{1}{\alpha^2} \int \Omega_{p+1}. \quad (\text{A.1})$$

Here $G_{\mu\nu}$ is the manifestly supersymmetric induced world-volume metric⁸

$$G_{\mu\nu} = \eta_{mn} \Pi_\mu^m \Pi_\nu^n, \quad \Pi_\mu^m = \partial_\mu X^m - \bar{\theta} \Gamma^m \partial_\mu \theta, \quad (\text{A.2})$$

and the Born-Infeld field strength $\mathcal{F}_{\mu\nu}$ is given by

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - b_{\mu\nu}, \quad b_{\mu\nu} = \alpha^{-1} \bar{\theta} \sigma_3 \Gamma_m \partial_\mu \theta \left(\partial_\nu X^m - \frac{1}{2} \bar{\theta} \Gamma^m \partial_\nu \theta \right) - (\mu \leftrightarrow \nu), \quad (\text{A.3})$$

where Ω_{p+1} is a particular $p + 1$ -form [35–37] (see e.g. (45) in [36]).

The action has the global supersymmetry

$$\begin{aligned} \delta_\epsilon \theta &= \epsilon, & \delta_\epsilon X^m &= -\bar{\theta} \Gamma^m \epsilon, \\ \delta_\epsilon A_\mu &= -\alpha^{-1} \bar{\theta} \Gamma_m \sigma_3 \epsilon \partial_\mu X^m + \frac{\alpha^{-1}}{6} (\bar{\theta} \sigma_3 \Gamma_m \epsilon \bar{\theta} \Gamma^m \partial_\mu \theta + \bar{\theta} \Gamma_m \epsilon \bar{\theta} \sigma_3 \Gamma^m \partial_\mu \theta). \end{aligned} \quad (\text{A.4})$$

Note that as a consequence of the transformations above we have that

$$\delta_\epsilon \mathcal{F} = 0. \quad (\text{A.5})$$

Also note that to show invariance of the action, we need the following $d = 10$ Fierz identity valid for any three Majorana-Weyl spinors $\lambda_1, \lambda_2, \lambda_3$ of the same chirality

$$\Gamma_m \lambda_1 \bar{\lambda}_2 \Gamma^m \lambda_3 + \Gamma_m \lambda_2 \bar{\lambda}_3 \Gamma^m \lambda_1 + \Gamma_m \lambda_3 \bar{\lambda}_1 \Gamma^m \lambda_2 = 0. \quad (\text{A.6})$$

Besides the global supersymmetry the action is also invariant under a local κ -symmetry given by

$$\begin{aligned} \delta_\kappa X^m &= \bar{\theta} \Gamma^m (\mathbb{1} + \Gamma) \kappa, & \delta_\kappa \theta &= (\mathbb{1} + \Gamma) \kappa, \\ \delta_\kappa A_\mu &= \alpha^{-1} \bar{\theta} \sigma_3 \Gamma_m \delta_\kappa \theta \partial_\mu X^m \\ &\quad - \frac{1}{2} \alpha^{-1} \bar{\theta} \sigma_3 \Gamma_m \delta_\kappa \theta \bar{\theta} \Gamma^m \partial_\mu \theta - \frac{1}{2} \alpha^{-1} \bar{\theta} \Gamma^m \delta_\kappa \theta \bar{\theta} \sigma_3 \Gamma_m \partial_\mu \theta, \end{aligned} \quad (\text{A.7})$$

⁸We use a doublet θ^I , $I = 1, 2$ of Majorana-Weyl spinors of the same chirality. σ_3 in (A.3) and below indicates a Pauli $(\sigma_3)_I{}^J$ matrix (or any other projection matrix). If it is clear by the context, we will omit the I index as well as any spinorial index.

where $\kappa(\sigma) \rightarrow \kappa^I(\sigma)$ is an arbitrary doublet of Majorana-Weyl spinors of the same chirality. Γ is a hermitian traceless product structure, *i.e.*, it satisfies

$$\text{Tr } \Gamma = 0, \quad \Gamma^2 = 1. \quad (\text{A.8})$$

The precise form of Γ can be found by explicitly imposing κ -symmetry in (A.1). In the usual Pauli matrices basis, and acting on positive-chirality spinors θ^I , Γ is given by

$$\Gamma = \begin{pmatrix} 0 & \beta_- \\ (-1)^n \beta_+ & 0 \end{pmatrix}. \quad (\text{A.9})$$

where β_+ and β_- are matrices that satisfy

$$\beta_- \beta_+ = \beta_+ \beta_- = (-1)^n. \quad (\text{A.10})$$

They are constructed using pull-backs of the matrices Γ^m to the world-volume:

$$\hat{\Gamma}_\mu \equiv \Pi_\mu^m \Gamma_m, \quad \hat{\Gamma}^\mu \equiv G^{\mu\nu} \hat{\Gamma}_\nu = \Pi_m^\mu \Gamma^m, \quad \Pi_m^\mu = G^{\mu\nu} \Pi_\nu^n \eta_{mn}, \quad (\text{A.11})$$

where $G^{\mu\nu}$ is the inverse of $G_{\mu\nu}$. They satisfy the Clifford algebra relations $\hat{\Gamma}^\mu \hat{\Gamma}^\nu + \hat{\Gamma}^\nu \hat{\Gamma}^\mu = 2G^{\mu\nu}$. We have

$$\Pi_m^\mu \Pi_\nu^m = \delta_\nu^\mu, \quad (\text{A.12})$$

but since Π is only a square matrix for $p = 9$, Π_m^μ is only the inverse matrix of Π_μ^m in that case.

In terms of the pull-backs, the matrices β_+ and β_- are given by

$$\begin{aligned} \beta_+ &\equiv \mathcal{G} se^{\frac{\alpha}{2} \mathcal{F}_{\mu\nu} \hat{\Gamma}^{\mu\nu}} \Gamma_{(0)}^{Dp} \equiv \mathcal{G} \sum_{k=0}^{n+1} \frac{\alpha^k}{2^k k!} \hat{\Gamma}^{\mu_1 \nu_1 \dots \mu_k \nu_k} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_k \nu_k} \Gamma_{(0)}^{Dp}, \\ \beta_- &\equiv \mathcal{G} se^{-\frac{\alpha}{2} \mathcal{F}_{\mu\nu} \hat{\Gamma}^{\mu\nu}} \Gamma_{(0)}^{Dp} \equiv \mathcal{G} \sum_{k=0}^{n+1} \frac{(-\alpha)^k}{2^k k!} \hat{\Gamma}^{\mu_1 \nu_1 \dots \mu_k \nu_k} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_k \nu_k} \Gamma_{(0)}^{Dp}. \end{aligned} \quad (\text{A.13})$$

with

$$\mathcal{G} = \frac{\sqrt{|G|}}{\sqrt{|G + \alpha \mathcal{F}|}} = [\det(\delta_\mu^\nu + \alpha \mathcal{F}_{\mu\rho} G^{\rho\nu})]^{-1/2}. \quad (\text{A.14})$$

Here *se* stands for the skew-exponential function (*i.e.* the exponential function with skew-symmetrized indices of the gamma matrices at every order in the expansion so the expansion has effectively only a finite number of terms). β_- is related to β_+ by replacing \mathcal{F} by $-\mathcal{F}$.

The matrix $\Gamma_{(0)}^{Dp}$ is defined by

$$\Gamma_{(0)}^{Dp} = \frac{1}{(p+1)! \sqrt{|G|}} \varepsilon^{\mu_1 \dots \mu_{p+1}} \hat{\Gamma}_{\mu_1 \dots \mu_{p+1}}, \quad (\Gamma_{(0)}^{Dp})^2 = (-1)^n. \quad (\text{A.15})$$

In case of $p = 9$ it agrees with the matrix defined in flat gamma matrices:

$$\Gamma_{(0)}^{D9} = \frac{1}{10! \sqrt{|G|}} \varepsilon^{\mu_0 \dots \mu_9} \hat{\Gamma}_{\mu_0 \dots \mu_9} = \frac{1}{10!} \varepsilon^{m_0 \dots m_9} \Gamma_{m_0 \dots m_9} = \Gamma_*, \quad (\text{A.16})$$

The theory is also invariant under general coordinate transformations $\sigma^{\mu'} = \sigma^\mu + \xi^\mu(\sigma)$ on the worldvolume. The complete set of transformations on the fields of the theory is hence given by supersymmetry transformations δ_ϵ , κ -transformations δ_κ , covariant general coordinate transformations on the worldvolume δ_ξ , and a $U(1)$ gauge transformation $\delta_{U(1)}$:

$$\begin{aligned}\delta\theta &= \delta_\epsilon\theta + \delta_\kappa\theta + \xi^\mu\partial_\mu\theta, \\ \delta X^m &= \delta_\epsilon X^m + \delta_\kappa X^m + \xi^\mu\partial_\mu X^m, \\ \delta A_\mu &= \delta_\epsilon A_\mu + \delta_\kappa A_\mu + \delta_{U(1)}A_\mu + \xi^\rho F_{\rho\mu}.\end{aligned}\tag{A.17}$$

A.2 Gauge-fixing the Dp-superbrane

Upon gauge fixing of κ -symmetry and general coordinate transformations, the global target space supersymmetry combines with a special field dependent κ -transformation into a global worldvolume supersymmetry. Writing

$$X^m = \{X^{m'}, \phi^I\}, \quad m' = 0, 1, \dots, p, \quad I = 1, \dots, 9-p,\tag{A.18}$$

where m' refers to the $p+1$ worldvolume directions and I refers to the $9-p$ transverse directions, we impose the following gauge-fixing conditions

$$(\mathbb{1} + \sigma_3)\theta = 0, \quad X^{m'} = \delta_\mu^{m'}\sigma^\mu.\tag{A.19}$$

Using the basis where σ_3 is the diagonal Pauli matrix, the condition (A.19) implies $\theta^1 = 0$. From now on we will use $\theta^2 \equiv \alpha\lambda$.

In this gauge the Wess-Zumino term Ω_{p+1} becomes constant (-1) and the action is given by

$$S = -\frac{1}{\alpha^2} \int d^{p+1}\sigma \left\{ \sqrt{-\det(G_{\mu\nu} + \alpha\mathcal{F}_{\mu\nu})} - 1 \right\},\tag{A.20}$$

together with

$$\begin{aligned}G_{\mu\nu} &= \eta_{m'n'}\Pi_\mu^{m'}\Pi_\nu^{n'} + \delta_{IJ}\Pi_\mu^I\Pi_\nu^J, \\ \Pi_\mu^{m'} &= \delta_\mu^{m'} - \alpha^2\bar{\lambda}\Gamma^{m'}\partial_\mu\lambda, \quad \Pi_\mu^I = \partial_\mu\phi^I - \alpha^2\bar{\lambda}\Gamma^I\partial_\mu\lambda,\end{aligned}\tag{A.21}$$

and

$$\begin{aligned}\mathcal{F}_{\mu\nu} &\equiv F_{\mu\nu} - b_{\mu\nu}, \\ b_{\mu\nu} &= -2\alpha\bar{\lambda}\Gamma_{[\nu}\partial_{\mu]}\lambda - 2\alpha\bar{\lambda}\Gamma_I\partial_{[\mu}\lambda\partial_{\nu]}\phi^I.\end{aligned}\tag{A.22}$$

In order to preserve the gauge-fixing conditions (A.19) we have to impose

$$\delta\theta^1 = 0, \quad \delta X^{m'} = 0,\tag{A.23}$$

from which we obtain

$$\epsilon^1 + \kappa^1 + \beta_- \kappa^2 = 0 \quad \text{and} \quad \xi^{m'} = \alpha \left\{ \bar{\lambda} \Gamma^{m'} \epsilon^2 + (-1)^n \bar{\lambda} \Gamma^{m'} \beta_+ \epsilon^1 \right\}, \quad (\text{A.24})$$

together with the corresponding gauge-fixed, non-linear realization of the remaining transformations

$$\begin{aligned} \delta_{\epsilon^{1,2}} \phi^I &= -\alpha \bar{\lambda} \Gamma^I \epsilon^2 - (-1)^n \alpha \bar{\lambda} \Gamma^I \beta_+ \epsilon^1 + \xi^\mu \partial_\mu \phi^I, \\ \delta_{\epsilon^{1,2}} \lambda &= \alpha^{-1} (\epsilon^2 + (-1)^{n+1} \beta_+ \epsilon^1) + \xi^\mu \partial_\mu \lambda, \\ \delta_{\epsilon^{1,2}} A_\mu &= \bar{\lambda} (\Gamma_\mu + \Gamma_I \partial_\mu \phi^I) (\epsilon^2 + (-1)^n \beta_+ \epsilon^1) \\ &\quad - \alpha^2 \bar{\lambda} \Gamma_m (\tfrac{1}{3} \epsilon^2 + (-1)^n \beta_+ \epsilon^1) \bar{\lambda} \Gamma^m \partial_\mu \lambda + \xi^\rho F_{\rho\mu}. \end{aligned} \quad (\text{A.25})$$

The above transformations are parametrized by two spinors ϵ^1 and ϵ^2 , and therefore we have two independent sets of sixteen supersymmetry transformations. The deformed linear symmetries and the VA-type non-linear ones are given by the change of basis

$$\epsilon^1 = -\tfrac{1}{2} i \Gamma_*^{(p)} \epsilon, \quad \epsilon^2 = -\tfrac{1}{2} \epsilon + \zeta. \quad (\text{A.26})$$

where $\Gamma_*^{(p)} = (-i)^n \Gamma^0 \dots \Gamma^p$ (and in particular $\Gamma_*^{(9)} = \Gamma_*$). Using this in (A.24) gives

$$\xi^m = \xi_\epsilon^m + \xi_\zeta^m, \quad \xi_\epsilon^m \equiv -\tfrac{1}{2} \alpha \bar{\lambda} \Gamma^m [\mathbb{1} + \beta] \epsilon, \quad \xi_\zeta^m = \alpha \bar{\lambda} \Gamma^m \zeta, \quad (\text{A.27})$$

where

$$\beta = (-i)^n \beta_+ \Gamma_*^{(p)} = (-i)^n \mathcal{G} \sum_{k=0}^{n+1} \frac{\alpha^k}{2^k k!} \hat{\Gamma}^{\mu_1 \nu_1 \dots \mu_k \nu_k} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_k \nu_k} \Gamma_{(0)}^{Dp} \Gamma_*^{(p)} = 1 + \mathcal{O}(\alpha), \quad (\text{A.28})$$

when terms with $\partial_\mu \phi^I$ are also considered as order α (see e.g. (3.13)). The corresponding transformations are

$$\begin{aligned} \delta_\epsilon \phi^I &= \tfrac{1}{2} \alpha \bar{\lambda} \Gamma^I [\mathbb{1} + \beta] \epsilon + \xi_\epsilon^\mu \partial_\mu \phi^I, \\ \delta_\epsilon \lambda &= -\tfrac{1}{2\alpha} [\mathbb{1} - \beta] \epsilon + \xi_\epsilon^\mu \partial_\mu \lambda, \\ \delta_\epsilon A_\mu &= -\tfrac{1}{2} \bar{\lambda} (\Gamma_\mu + \Gamma_I \partial_\mu \phi^I) [\mathbb{1} + \beta] \epsilon \\ &\quad + \tfrac{1}{2} \alpha^2 \bar{\lambda} \Gamma_m [\tfrac{1}{3} \mathbb{1} + \beta] \epsilon \bar{\lambda} \Gamma^m \partial_\mu \lambda + \xi_\epsilon^\rho F_{\rho\mu}. \end{aligned} \quad (\text{A.29})$$

and

$$\begin{aligned} \delta_\zeta \phi^I &= -\alpha \bar{\lambda} \Gamma^I \zeta + \xi_\zeta^\mu \partial_\mu \phi^I, \\ \delta_\zeta \lambda &= \alpha^{-1} \zeta + \xi_\zeta^\mu \partial_\mu \lambda, \\ \delta_\zeta A_\mu &= \bar{\lambda} (\Gamma_\mu + \Gamma_I \partial_\mu \phi^I) \zeta - \tfrac{1}{3} \alpha^2 \bar{\lambda} \Gamma_m \zeta \bar{\lambda} \Gamma^m \partial_\mu \lambda + \xi_\zeta^\rho F_{\rho\mu}. \end{aligned} \quad (\text{A.30})$$

B Appendix: Vp-branes

B.1 Vector p-branes in $d = 6$

In the previous appendix we constructed supersymmetric Born-Infeld actions with 16+16 supersymmetries corresponding to Dp-branes in $d = 10$. It is the purpose of this section to give similar results for Vp-branes with 8+8 supersymmetries in $d = 6$. We remind that Vp-branes are branes whose worldvolume content is given by a vector multiplet but whose tension is not necessarily proportional to the inverse string coupling constant. Since the discussion in this appendix is rather similar to the previous appendix we will only highlight a few relevant formulae.

The κ -symmetric Vp-brane action (with $p = 2n + 1$ odd), in a flat background geometry with (longitudinal and transverse) coordinates X^m , $m = 0, \dots, 5$, consists of the Dirac-Born-Infeld-Nambu-Goto term S_{DBI} and Wess-Zumino term S_{WZ} and is formally completely the same as (A.1). All formulas of Sec. A.1 are still applicable.⁹

To show invariance of the action, we need in this case the $d = 6$ Fierz identity valid for any three symplectic Majorana-Weyl spinors $\lambda_1^i, \lambda_2^j, \lambda_3^k$ of the same chirality, that is also formally the same as (A.6), or explicitly

$$\Gamma_m \lambda_1^i \bar{\lambda}_2^j \Gamma^m \lambda_3^k \varepsilon_{kj} + \Gamma_m \lambda_2^i \bar{\lambda}_3^j \Gamma^m \lambda_1^k \varepsilon_{kj} + \Gamma_m \lambda_3^i \bar{\lambda}_1^j \Gamma^m \lambda_2^k \varepsilon_{kj} = 0. \quad (\text{B.1})$$

Also the global ϵ and local κ -supersymmetry have the same form as in (A.4) and (A.7), where $\kappa(\sigma) \rightarrow \kappa^{Ii}(\sigma)$ is an arbitrary doublet of symplectic Majorana-Weyl spinors of the same chirality, and the Γ^m and Π_μ^m are, of course, the $d = 6$ quantities.

The theory is also invariant under general coordinate transformations $\sigma^{\mu'} = \sigma^\mu + \xi^\mu(\sigma)$ on the worldvolume. Hence, just as in (A.17) of the previous appendix, the complete set of transformations on the fields of the theory is given by supersymmetry transformations, κ -transformations, general coordinate transformations on the worldvolume and a $U(1)$ gauge transformation.

B.2 Gauge fixing

The gauge fixing of κ -symmetry and general coordinate transformations goes exactly as in Sec. A.2, and the formulas remain applicable, with the understanding that now $I = 1, \dots, 5 - p$.

The decomposition rules and non-linear gauge-fixed realization of the remaining transformations is analogous as in (A.24) and (A.25). Also the split into two independent sets of, in this case eight,

⁹In this case, we use a doublet θ^{Ii} , $I = 1, 2$ of symplectic Majorana-Weyl spinors of the same chirality. The symplectic indices i are implicitly present in the same way as the index I . In a bilinear we assume that they are contracted using the antisymmetric ε_{ij} , e.g. $\bar{\lambda} \Gamma^m \chi \equiv \bar{\lambda}^i \Gamma^m \chi^j \varepsilon_{ji}$.

supersymmetry transformations, is exactly as in (A.29) and (A.30).

C Notation

We follow the notation of [53], especially all coefficients in Appendix A in that book are $s_i = 1$ except for the normalization of the ϵ -supersymmetry, which is such that $s_9 = -2$. One can go to the standard notations of the book by multiplying all barred spinors by $-1/2$, e.g. the standard kinetic Lagrangian for the real fermion becomes $-\frac{1}{2}\bar{\lambda}\not{\partial}\lambda$ rather than $\bar{\lambda}\not{\partial}\lambda$. The γ -matrices for $d = 10$ or $d = 6$ are denoted as Γ^m . Note that the matrices Γ^m and Γ_μ are the flat $d = 10$ matrices, whereas $\hat{\Gamma}_\mu$ is defined in (A.11) as the pull-back to the world-volume:

$$\Gamma_\mu = \delta_\mu^m \Gamma_m, \quad \hat{\Gamma}_\mu = \Pi_\mu^m \Gamma_m = (\delta_\mu^m - \alpha^2 \bar{\lambda} \Gamma^m \partial_\mu \lambda) \Gamma_m. \quad (\text{C.1})$$

We use shortcuts for index contractions, such that $F^2 = F_{\mu\nu} F^{\mu\nu}$ and $\Gamma \cdot F = \Gamma^{\mu\nu} F_{\mu\nu}$. However, matrices F^3, \dots are defined as $(F^3)_{\mu\nu} = F_{\mu\rho} F^{\rho\sigma} F_{\sigma\nu}$, and $\text{Tr } F^4 = F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}$.

As in [53], we define

$$\Gamma_* = (-i)^{(d-2)/2} \Gamma^0 \dots \Gamma^{d-1}. \quad (\text{C.2})$$

The spinors for $d = 10$ and $d = 6$ are left-handed, which means that

$$\Gamma_* \lambda = \lambda, \quad \bar{\lambda} \Gamma_* = -\bar{\lambda}, \quad (\text{C.3})$$

$$d = 10 : \Gamma_* = \Gamma^0 \Gamma^1 \dots \Gamma^9, \quad d = 6 : \Gamma_* = -\Gamma^0 \Gamma^1 \dots \Gamma^5.$$

In this work appear Majorana and symplectic-Majorana spinors that satisfy the Majorana flip relations

$$\begin{aligned} \bar{\lambda}_1 \lambda_2 &= \bar{\lambda}_2 \lambda_1, & \bar{\lambda}_1 \Gamma^\mu \lambda_2 &= -\bar{\lambda}_2 \Gamma^\mu \lambda_1, & \bar{\lambda}_1 \Gamma^{\mu\nu} \lambda_2 &= -\bar{\lambda}_2 \Gamma^{\mu\nu} \lambda_1, & \bar{\lambda}_1 \Gamma^{\mu\nu\rho} \lambda_2 &= \bar{\lambda}_2 \Gamma^{\mu\nu\rho} \lambda_1, \\ \bar{\lambda}_1 \Gamma^{\mu_1 \dots \mu_r} \lambda_2 &= (-)^{r(r+1)/2} \bar{\lambda}_2 \Gamma^{\mu_1 \dots \mu_r} \lambda_1 \end{aligned} \quad (\text{C.4})$$

For $d = 6$ this is accomplished due to a definition where the indices $i, j, \dots = 1, 2$ are hidden and

$$\bar{\lambda} \Gamma^\mu \chi \equiv \bar{\lambda}^i \Gamma^\mu \chi^j \epsilon_{ji}. \quad (\text{C.5})$$

In this way the formulas can be used for all dimensions $d = 2, 3, 4, 6$ and 10 , where also the cyclic identity (A.6) holds, which for $d = 6$ is given explicitly in (B.1).

To reduce the $d = 10$ expressions to $d = 4$ spinors in Sec. 3.2, we construct the 32×32 matrices

Γ^m , with $m = \mu, a, a+3$ ($\mu = 0, \dots, 3$ and $a = 1, 2, 3$) from 4×4 matrices γ^μ by

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu \otimes \mathbb{1}_8, & \Gamma^a &= \gamma_* \otimes \begin{pmatrix} 0 & \beta^a \\ -\beta^a & 0 \end{pmatrix}, & \Gamma^{a+3} &= \gamma_* \otimes \begin{pmatrix} 0 & i\alpha^a \\ i\alpha^a & 0 \end{pmatrix}, \\ C_{10} &= C_4 \otimes \begin{pmatrix} 0 & \mathbb{1}_4 \\ \mathbb{1}_4 & 0 \end{pmatrix}, & \Gamma_* &= \gamma_* \otimes \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -\mathbb{1}_4 \end{pmatrix},\end{aligned}\tag{C.6}$$

where C_{10} is the charge conjugate in 10 dimensions, and C_4 the one of $d = 4$. Here we use the 4×4 antisymmetric real matrices α^a and β^a from [44, 45], whose components are indicated by indices $i, j = 1, \dots, 4$, and which satisfy

$$\begin{aligned}\{\alpha^a, \alpha^b\} &= \{\beta^a, \beta^b\} = -2\delta^{ab}, & [\alpha^a, \beta^b] &= 0, \\ \alpha_{ij}^a \alpha_{kl}^a &= 2\delta_{i[k} \delta_{\ell]j} + \varepsilon_{ijkl}, & \beta_{ij}^a \beta_{kl}^a &= 2\delta_{i[k} \delta_{\ell]j} - \varepsilon_{ijkl}, \\ \alpha_{ij}^a &= \frac{1}{2}\varepsilon_{ijkl} \alpha_{kl}^a, & \beta_{ij}^a &= -\frac{1}{2}\varepsilon_{ijkl} \beta_{kl}^a, & \alpha_{ij}^a \alpha^{bij} &= \beta_{ij}^a \beta^{bij} = 4\delta^{ab}, \\ \alpha^1 \alpha^2 \alpha^3 &= \mathbb{1}_4, & \beta^1 \beta^2 \beta^3 &= -\mathbb{1}_4.\end{aligned}\tag{C.7}$$

The engineering dimensions that are used for the various fields in a d -dimensional action are

$$\begin{aligned}[x] &= -1, & [\theta, \epsilon, \eta, \zeta, \kappa] &= -1/2, & [A_\mu] &= d/2 - 1, & [F_{\mu\nu}] &= d/2, & [\lambda] &= (d-1)/2, \\ [\alpha] &= -d/2, & [\partial_\mu] &= 1, & [X] &= -1, & [b_{\mu\nu}] &= d/2, & [\phi^I] &= -1.\end{aligned}\tag{C.8}$$

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